MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

13 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 8 printed pages (inclusive of the front page) with 3 questions only. Page 6 TO 8 are appendices for the rules of inference, logical equivalence laws and Standard Normal distribution table.
2. Attempt **ALL THREE** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.
4. Show **ALL** of your working steps clearly.

QUESTION 1 **[TOTAL: 17 MARKS]**

a) Let the universal set $U = \{0, 1, 2, \cdots, 10\}$, $A = \{x \mid x \text{ is a prime number}\}$ $B = \{1, 2, 3, 4, 9\}$, and $C = \{x \mid x \text{ is even}\}$.

i) List all the elements of A and C. [1 mark]

ii) Find $A \cap B$ and $B - \overline{C}$. [2 marks]

iii) Find $\mathrm{P}(A \cap B)$ [1 mark]

b) Let S be the relation defined on the set $A = \{1, 4, 5\}$ such that

$$S = \{(a,b) \in A \times A \mid (a-b) \text{ divides } 18\}$$

i) List all elements of $A \times A$. [1 mark]

ii) List all elements of S. [1 mark]

iii) Is S transitive? Justify your answer. [1 mark]

iv) Is S a function? Justify your answer. [1 mark]

c) Use **truth table** to show that propositions $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow p \vee r$ are logically equivalence. [3 marks]

d) Suppose an argument consisting of the following premises and conclusion:
Premises:

$\exists x(\neg R(x)), \forall x(P(x) \vee Q(x))$, $\forall x(Q(x) \rightarrow S(x))$, $\forall x(\neg(\neg R(x) \wedge S(x)))$

Conclusion:

$\exists x P(x)$

Use **rules of inference** to show that the argument is true. Please state clearly each of the law that you used in your proof. [6 marks]

Continued ...

QUESTION 2 [TOTAL: 17 MARKS]

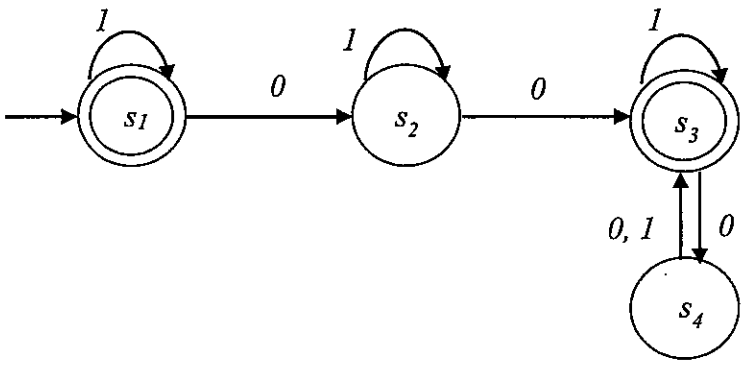
a) Given the recursion

$$r_0 = 1, \quad r_1 = 7,$$
$$r_n = 2r_{n-1} + 3r_{n-2}, \quad n \geq 2.$$

Use the strong induction to proof that $r_n = (-1)^{n+1} + 2(3^n)$ for every integer $n \geq 0$, is the explicit formula for the above recursion. [5 marks]

b) Show that $f(n) = 3n^2 + \frac{1}{2}n + 1$ is $\Theta(n^2)$. [3 marks]

c) Let A be the finite-state automaton with sets of finite states $S = \{s_1, s_2, s_3, s_4\}$ and input values $I = \{0, 1\}$. The initial state is s_1 and the accepting states are s_1 and s_3. The state transition diagram of A is given as follows.



i) Give a bit string of length four that is accepted by A. Give a bit string of length four that is not accepted by A. [1 mark]

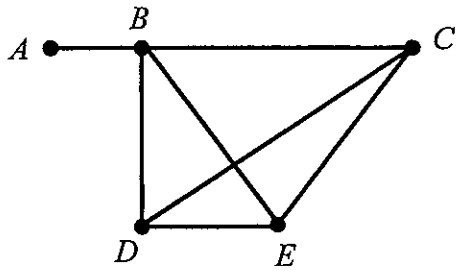
ii) Show the state transitions of the automaton A when input string *01001* is given. Is automaton A accepting the input string *01001*? [2 marks]

iii) Use regular expression to express the language that is accepted by the automaton A. [1 mark]

Continued ...

QUESTION 2 (CONTINUED) **[TOTAL: 17 MARKS]**

d) Given the following graph:



i) Use the depth first search algorithm to find a spanning tree in the above graph. Assume vertex B is the root and the vertices are ordered alphabetically. Show clearly each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the stack for the search algorithm and the third column is the edges of the resulting spanning tree. [4 marks]

ii) Find the Hamiltonian circuit for the above graph if it exists or explain why it doesn't have one. [1 mark]

Continued ...

QUESTION 3 **[TOTAL: 16 MARKS]**

[LEAVE YOUR ANSWERS IN SIMPLEST FRACTION FORM OR CORRECT TO TWO DECIMAL PLACES.]

a) There are 50 baskets of peaches. Each basket contains no more than k peaches. Find the value of k if there are at least 3 baskets containing the same number of peaches. [2 marks]

b) In a magic show, there are 10 adults and 8 kids as audiences. The magician wants to select 5 audiences to participate in his magic show.

i) Find the number of ways of selecting the 5 audiences. [1 mark]

ii) What is the probability that there is at least 2 adults and at least 2 kids from the audience selection?

[1.5 marks]

c) A random variable X describes the number of day(s) that students absent from TMA1201 classes in a particular month. The following table summarizes the probability mass function (pmf) of the X.

x	0	1	2	3	4
$p(x)$	0.5	a	$2a$	0.15	0.05

Find the

i) value of a. [1.5 marks]

ii) probability that a student is absent no more than 2 days in a particular month. [1 mark]

iii) expected value and variance for X. [3 marks]

d) Let X be a random variable that explains the number of girls in 3 birth and it follows Binomial distribution. Find the probability to get 2 girls in 3 birth. [2 marks]

e) The heights of female teenagers are normally distributed with a mean of 167.68cm and standard deviation of σ cm. It is known that 5% of the female teenagers are shorter than 154cm.

i) Find the value of σ. [2 marks]

ii) It is found that 23% of female teenagers are taller than k cm. Find the value of k. [2 marks]

Continued ...

Appendix

List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Conversion of Equivalence: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Double Negation: $\neg \neg p \Leftrightarrow p$

DeMorgan : (i) $\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(ii) $\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination : (i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Negation : (i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

Identity : (i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Commutative : (i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Idempotent : (i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Distributive : (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Associative : (i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$

(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

Absorption : (i) $p \vee (p \wedge q) \Leftrightarrow p$

(ii) $p \wedge (p \vee q) \Leftrightarrow p$

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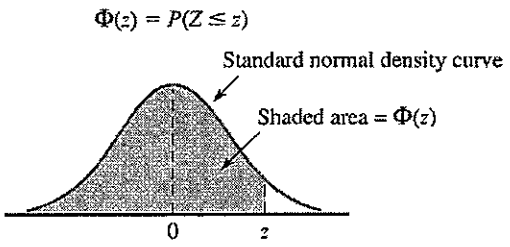
Appendix II Standard Normal Curve Areas

Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Continued ...

Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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